Final Exam Cosmology - Study year 2022-2023

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The exam duration is 2 hours.
The grade is out of 100 and will be based on your answers to all the questions in the exam.
No external material is allowed, except the formula sheer provided to you in the exam and a calculator.
The number of points given to each question is indicated next to it.
Indicate clearly the steps in your solution and provide sufficient text.
If you are unable to answer a subquestion on which a later question depends, please answer the later question by outlining the method clearly without plugging in the number from the previous one.

I. This part of the exam is concerned with the geometry of the Universe and its evolution with time as described by Friedmann's equation. Please answer all 7 subquestions. The total number of points for this question is 50.

1. **Explain why a flat Universe ($\kappa = 0$) with matter, radiation and vacuum energy, is asymptotically dominated by the vacuum energy component (i.e., $\lim_{t\to\infty} \Omega = \Omega_\Lambda$).** (5 points)

2. **Show that a curved Universe ($\kappa \neq 0$) with a vacuum energy component, Ω_Λ is asymptotically flat. Discuss what the relation of this feature to inflation.** (5 points)

Now, consider the case of a Universe and in which the expansion factor as a function of t follows the semicircle $a/a_0 = A\sqrt{1-\left(\frac{t}{t_m}-1\right)^2}$, where t_m is time at which the scale factor is maximum and A is a dimensionless constant. Assume that $a_0 = 1$ at the current time t_0 and that $t_0 = \frac{t_m}{2}$ (t_m is given).

3. **Calculate the value of A.** (5 points)

4. **Express the cosmic time, t, as a function of the scale factor, a, and then as a function of the redshift, z, in this model for the universe.** (10 points)

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5. Use the FLRW metric to show that the distance a photon has traveled to reach us is $d_{photon}(t) = \int_t^{t_0} \frac{dt}{a(t)}$. Use this expression to measure distance the photons from a galaxy as a function of redshift. Remember that photons from this galaxy follow the null geodesic. Hint: $\int \frac{d\xi}{\sqrt{1-\xi^2}} = \arcsin \xi$ (10 points)

6. Calculate H^2, i.e., $(\dot{a}/a)^2$, as a function of the scale factor a. What type of Universe is this and what are its current energy density values in terms of the critical density? (15 points)

II. For this question we will assume that the Universe is flat and matter dominated throughout its history ($\Omega_{m,0} = 1$). Assume also the Hubble constant today is 70 km/s/Mpc and that all the baryons in the Universe are protons (i.e., no neutrons, helium or more massive atoms exist). The total number of points for this question is 50

1. In order to describe recombination we used the Saha equation. Answer the following two questions about this equation:
   - Why the factor $\frac{1}{\hbar^3}$ appears in the equation? (5 points)
   - What are the values of the statistical weights (g's) the appear in the equation for recombination? Explain why. (5 points)

2. Before recombination at redshift 1100 the Universe was highly ionized. Calculate the mean free path for Thomson scattering at that redshift assuming the Universe was still highly ionized and compare it to Hubble radius at that redshift. (5 points)

3. As mentioned in the class the Universe currently is highly ionized. Calculate the mean free path for Thomson scattering at $z = 0$ and compare it to the current Hubble radius. (5 points)

4. In the real Universe the process of decoupling of photons from electrons occurs after the recombination of electrons and protons to form neutral hydrogen. How do we know that recombination occurred? (5 points).

5. Assuming that the recombination process never happened in the Universe (it is always ionized) at what redshift then the decoupling of photons from electrons would occur? Remember we are still assuming a flat and matter dominated Universe. (15 points)

6. What is the event horizon distance at that redshift? (10 points)

Good Luck!